SRT MEMO #020

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
HAYSTACK OBSERVATORY
WESTFORD, MASSACHUSETTS 01886

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Telephone: 978-692-4764
Fax: 781-981-0590

To: SRT Group

From: Alan E.E. Rogers

Subject: VLBI data signal analysis

This memo summarizes the signal analysis.

1] Geometry calculations

Each second the Apriori baseline delay and delay rate are calculated. The delay for each VLBI data block is calculated for each VLBI data block using the rate to interpolate the delay between seconds. Each block is 512 bytes (4096 samples) of 512 microseconds duration.

2] Data correlation

The data is correlated with an Apriori offset which is the nearest integral number of 125 nanoseconds sample: shift = (delay/125) where τ is in nanoseconds

$$\rho_{xy}(\tau) = (1/N) \sum_{t=0}^{t=N-1} x(t)\, y(t + shift - \tau)$$

The first 8 bytes of the data are excluded from the correlation because they contain block ID and total power information.

3] Clipping correlation

The quantized data correlation is converted to an estimate of the true correlation using the "VanVleck" clipping correction

$$R_{xy}(\tau) = \sin\left((\pi/2)\,\rho_{xy}(\tau)\right)$$

4] Cross-spectral function

The correlation function is transformed to the cross-spectral function

$$S'_{xy}(w) = S_{xy}(w)\, e^{-i(w\Gamma + \phi)}$$

where $w\Gamma = 2\pi kF/M$

M = size of FFT used to get cross-spectrum (256)
F = fractional delay in units of the sample period (known in VLBI as the "fractional bit")
ϕ = fringe rotation phase in radians

For the upper sideband system of the SRT

$$\phi = (2\pi) \times delay(\text{secs}) \times frequency(Hz)$$

6] Coherence averaging of the cross-spectrum

Following the "fractional bit" correction and fringe rotation the cross-spectrum can be averaged for a period up to the expected coherence time.

$$S_{av} = \sum_{0}^{M/2} S$$

We only need to average the positive frequencies, since the negative frequencies are redundant and are not used.

7] Estimation of the normalized correlation amplitude and residual delay and phase we now transform the cross-spectrum back to form a "delay function" using a larger FFT to avoid losses in the interpolation of the amplitude, delay and phase when the peak falls between output points. A doubling of the FFT size to 2M is adequate if parabolic interpolation is used between points. The delay function is $D(\tau) = \sum_{0}^{M-1} S_{xy}(w) e^{i2\pi w\tau/(2M)}$.

The interpolated amplitude A, delay d and phase ϕ are

$$d = (k + F) \times 62.5 \quad ns$$

$$\phi = a\tan 2(\operatorname{Im} D_k, \operatorname{Re} D_k) - (\pi/4)F \quad radians$$

$$A = \left(\frac{2}{MN}\right)\left[p_k + (p_{k+1} - p_{k-1})F/4\right]^{1/2}$$

where $F = (p_{k+1} - p_{k-1})/(4p_k - 2p_{k-1} - 2p_{k+1})$

$p_k = |D|^2$

k = index for which $|D|^2$ is a maximum

N = number of blocks in coherent integration period.

References:
1] VanVleck, J.H., Middleton, D., *Proc IEEE*, **54**, 2, 1966.
2] Rogers, A.E.E., *in Methods of Experimental Physics*, **12**, part c, edited by M.L. Meeks, Academic Press, New York, 1976.
3] Thompson, A.R., Moran, J.M. and Swenson, G.W., *Interferometry and Synthesis in Radio Astronomy*, Wiley, New York, 2001.